



ENHANCING COLLEGE OF EDUCATION STUDENTS' UNDERSTANDING OF BASIC TRIGONOMETRY USING CONCRETE MANIPULATIVES

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Article Received 20-06-2020 , Accepted 18-07-2020 , Published 24-07-2020

Abstract

The paper aimed at examining the effect of concrete manipulative in the teaching and learning of basic trigonometry. In the study, some students were taught using the conventional technique and others taught with a manipulative approach. The study employed a sample of 140 College of Education students. A well structured, logically and systematically lessons with the use of manipulatives and other relevant materials such as students' worksheets was used. A pre-test was employed to assess the academic ability equivalence and homogeneity of the two groups, while post-testing was used to examine the effect of concrete manipulatives on the performance of students in trigonometry. The paper used a nonparametric test of the Mann-Whitney and Wilcoxon test to compare the mean values between the experimental and control groups to evaluate the data at an alpha level of 0.05. The finding of the study demonstrated no statistically significance between the two groups in the pretest. In the

posttest scores between the control and experimental groups, Mann-Whitney and Wilcoxon non-parametric test found a statistically significant difference ($p < 0.05, z = -8.360$). The study reiterates the interest of students revived from the use appropriate teaching aids, methodology, and an enabling classroom environment and consequently would make them appreciate fundamental trigonometry.

Keywords: Manipulatives, Trigonometry, Learning, College of Education

Introduction

Trigonometry is a significant study area, which is a pre-requisite to study a number of other topics in mathematics and other fields of study (bearings, global mathematics, vectors, mensuration, surveying, architecture, etc.). In view of this, trigonometry in most countries, especially in a developing country like Ghana is taught at all levels of education. Although this topic is useful in mathematics and other disciplines, most students are poorly attuned to it, and

therefore creates anxiety for students, with mention of the topic itself (Mensah, 2017). In addition, the ineffectiveness of instructional strategies used by some teachers in the field of mathematics in their classes for instructional delivery can be attributed to the poor performance of students in trigonometry both in internal and external examinations (Mitchell & Sutherland, 2020). According to the Ghana Education Service (GES), a lack of interest in strategic and conceptual teaching methodologies could be attributed to the poor performance of Ghanaian mathematics students (GES, 2018). There have been a variety of instructional strategies adopted in Ghana to improve mathematics performance at all levels of education, but many students are still finding mathematics daunting given the revised curriculum (Bruce, 2016).

Basic Trigonometry has been a challenge to most students especially at the pre-tertiary level in Ghana (Sakyi, 2014). The conceptions of fundamental trigonometry are difficult for most teachers in particular at the basic level where the foundations are needed for the students to understand the concept of trigonometry (Weber, Mejía-Ramos, Fukawa-Connelly & Wasserman, 2020). Some students find it difficult to identify basic trigonometric ratios from a right-angled triangle as well as solving simple trigonometric problems (Bosson-Amedenu, 2017). Students see trigonometry as something abstract and hence intend to avoid any question(s) relating to it. However, when simple trigonometry is taught using concrete manipulatives and linked real life, the concepts become more realistic in the students' minds. (Brahier, 2020). In the West African Secondary Schools Certificate Examination (WASSCE) conducted annually by the West African Examination Council, the poor performance among Senior High School (SHS) students is evident. According to the report from the chief examiner of the

West African Examination Council, consistently for the past nine years, most students have been avoiding trigonometric problems, bearings and other questions that demand the application of ideas from trigonometry (Council, 2019). In addition, the yearly report from the chief examiner also indicated that, the small proportion of candidates who attempt questions relating to trigonometry often fumble and score low marks (Council, 2019). This may be a big explanation for students' poor mathematical performance at WASSCE. Consequently, most students terminate their education after the WASSCE since core mathematics is a pre-requisite to the tertiary level. The consistent poor performance of students in relation to trigonometry calls for serious intervention programme(s) in order to improve the performance of students in trigonometry across all levels of education. It has been established that good intervention should provide strategies that are necessary for learning (Mitchell & Sutherland, 2020). Through organized activities, students are able to explore, explain, develop and measure their progress. (Hatfield & Chomitz, 2015).

The main elements in successful mathematical training are the promotion of strategic competence through practical activities, as asserted by Shellard and Moyer (2002). As students learn by using motivational approaches, they engage and challenge one another and thus enhance learning outcomes (Bryant, Bryant & Smith, 2019). Artzt, Armour-Thomas and Curcio (2008) reiterated this by their report, which concluded that the interactions between the students help each other and challenge each other's creative thinking. According to Protheroe (2007), students can work actively in mathematics, solve difficulties, share mathematical concepts, communicate mathematics with multiple representations and, in particular, employ manipulation

instruments and other materials. All these facts as suggested by Protheroe(2007) are missing in the conventional classroom(where the teacher teaches the topic in abstract terms and provide students with classroom exercises). The purpose of this study was to explore the effect of concrete manipulatives on the understanding of basic trigonometry by students.

The Concept of Trigonometry

As a physical branch of mathematics, trigonometry focuses on understanding the principles and their applications. Its contents include angles, angle measurements, triangles and their linkages (Rizkianto, Zulkardi, & Darmawijaya, 2013; Ahmad, Al Yakin & Sarbi, 2018; Ikeda & Stephens, 2020). It combines geometrical, graphic, and algebraic arguments which make sense in solving problems with triangles, trigonometric expressions, and graphs. Trigonometry is a widely used spatial and astronomical technology in many areas, such as electricity, cartography, geometry, maritime, optical and physical studies (Tuna, & Kacar, 2013). Trigonometry is one of six content domain fields listed in the Senior High School (SHS) Mathematics Curriculum in Ghana. It includes trigonometric principles, procedures and applications for the resolution of problems (Ministry of Education, 2010). Conceptual knowledge of trigonometrical ideas at the SHS level provides the basis for practical and meaningful mathematical learning at colleges of education.

In the College of Education (CoE) curriculum, the contents of trigonometric elements include trigonometric angle ratios, inverse trigonometric functions, bearings, maximum and minimum trigonometric functions and their graphs, solutions for basic trigonometric equations, trigonometric identities, compound angles; $\sin(A \pm B)$, $\cos(A \pm B)$, $\tan(A \pm B)$; and their

applications (Ministry of Education, 2007). The CoE curriculum program underlines constructivist practices, such as group study, project activity, debate, and discovery learning, recognizing the student will understand, make sense and benefit entirely from many advantages of trigonometry. It also emphasizes the advancement of ideas through concrete materials-based functional exercises in the development of cognitive skills for effective problem solving.

The Concept of Manipulatives

The word manipulative is synonymous to words like apparatus, tools, teaching aids, teaching materials, and learning materials. Yeatts (1991) describes manipulatives as objects that students can sense, handle, control and pass around. Students' senses are activated when students touch the manipulative objects, push them around, rearrange them and/or see them in various shapes and groupings. Manipulatives can also be defined as specific mathematical devices or artifacts designed to communicate and concretely represent abstract mathematical ideas(Moyer, 2001). Manipulatives are typically both virtual and real objects used for visualizing, abstracting ideas and facilitating learning. These are designed primarily for hands-on manipulation, and are ideal for teachers and students to use as models, have visual and tactile appeals.

Manipulations in any form, particularly in mathematics, must be used for teaching at all levels of education. Manipulative approaches can be called teaching aids, learning aids, instruments, equipment, educational assistance etc. Olayinka (2016) defines manipulative material as objects or devices which enable a teacher to make learnings meaningful for the teacher. Using manipulative methods can in general greatly improve the education of any subject in the school curriculum. According

to Wales (1975) knowledge discovered by students through the use of manipulatives stays with them for long as compared to the one impacted by the teacher. Adipo (2015) added that if a teacher uses appropriate manipulative to support his or her teaching, it enhances the students' innovative and creative thinking and consequently helps them to become more enthusiastic in learning the subject.

Instructional approaches to help manipulation should be adapted to boost the achievement of student mathematics (Gurbuz, Catlioglu, Birgin & Erdem, 2010).). Mathematical concepts using manipulatives techniques were described as a technique that enables students to draw on their own practical knowledge. By contrast, research has shown that institutions with ample teaching and learning facilities, a favorable student-teacher relationship, good workload and good rewards and incentives typically do better than those institutions lacking in these aspects (Brudett & Smith, 2003). Nevertheless, the use of teaching and learning resources is another perspective. Orji and Abolarin (2012) states that it is not appropriate to use manipulatives if the students are intelligent and the teacher has good knowledge. Haron, et al. (2019) argued that the key emphasis is to include students in the classroom activities. Not only do manipulatives boost the cognitive level of students, but they also increase their psychomotor ability (Cope, 2015; Kontas, 2016). The use of manipulatives should not be regarded as a solution to the difficulties of mathematical learning by students but should rather mean that the manipulation is useful for both teachers and learners (Kontas, 2016). If that is taken into account, manipulation tools can be used simply as a method for entertainment and nothing else at the end of an instructional delivery. Consequently, educators need to know when, why and how manipulative measures

should be used. Although the use of manipulatives has shown that the learning of mathematics by students is improved, there are challenges that can impede their effectiveness in instructional delivery. This makes them tend to use manipulatives in the conventional way of learning. Such problems include a lack of expertise of teachers in the use of manipulatives, manipulatives costs and lessons length (Holmes, 2013). The use of manipulatives can sometimes lead to cognitive uncertainty among some students (Clements, 2000). Such problems contribute to the ineffective use of manipulatives at higher education for exploitation. The student is believed to have established his logical ability to think abstractly at this level already (McNeil & Jarvin, 2007). There is no conclusion in existing literature about the use of mathematical manipulative materials and students' achievements. This is especially the case among higher education students. This is because these students are thought to have developed their abstract thought skills already. There is little among students at the higher level of education and this study therefore attempted to fill in gap on the use of manipulatives at the tertiary level of education.

Theoretical Perspective

The Action-Process-Object-Schema (APOS) Theory of Asiala, Brown and DeVries (1996) guided the study in an attempt to understand the instructional paradigm for students' understanding of the trigonometry. The APOS Theory aims to explain student knowledge-construction. According to the APOS principle, a student must have simple logical constructions in order to grasp a particular mathematical concept. These mental constructs are the most common actions, procedures, objects and schemes required to understand a concept. Theories require detection of the probable mental structures and the relevant learning activities

to aid the construction and reconstruction of these mental structures for certain concepts. The APOS theory is established in two main categories and in its application to classroom practice, according to Dubinsky (2010): (1) The capacity of the person to address mathematical issues in perceived mathematical problems and their solutions is by building or reconstructing mental structures; and (2) a person does not learn mathematical concepts directly, but rather applies the mental structures to understand concepts or situations. This facilitates the learning of trigonometry if a person has the necessary mental structures for the concepts. In the absence of mental structures, the response to conceptual problems, the understanding of trigonometric concepts and the application of trigonometry to reality can almost be impossible.

Research Methodology

A quasi-experimental design was employed as the research design for the study. The quasi experimental design is a research design which does not satisfy the most stringent external or internal validity criteria, for example with a generalization limited or with a design control over only one variable. The quasi experimental study design was used because it removes the problem of directionality and the independent variable is manipulated. However, because this does not involve random assignment of conditions, it does not eliminate the problem of confounding variables. The conception is depicted in Figure 1.

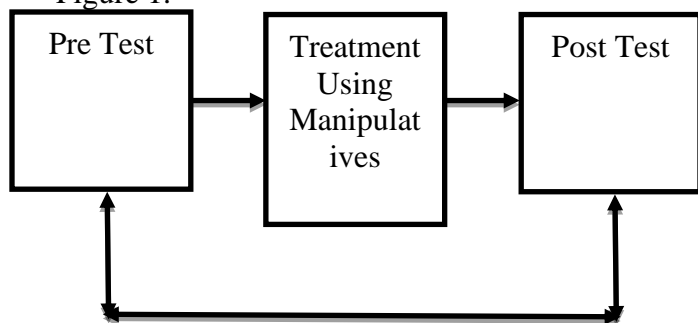


Figure 1: Experimental Design

This experimental study was conducted for a period of four week. The pre-intervention test (pretest) was conducted on the first day of instruction and the post-intervention test (Posttest) was conducted on the last day of instruction. The study sample was made up of all the first year students offering Basic of Education Degree Program at Bia-Lamplighter College of Education. This consisted of 56 females and 84 males; with mean age 23 ± 3.4 years.

In order to test learning about basic trigonometry, a Teacher Made Test (TMT) was performed. In order to address problems on the right-angled triangle and real-life issues the students are asked for identifying right-angled triangles, identified basic trigonometric ratios, and used simple trigonometric ratios to solve real life problems at pretest stage of the study. A printed question paper and an answer booklet were given to each student and test lasted for 60 minutes. After the experiment, a post-test was conducted to assess the efficacy of the two approaches. In the post-test, the researchers used the same questions used in in the pre-test. All the examination conditions and instructions used for the pretest were also adopted for the posttest with all the students taking part in both the pretest and posttest.

The independent variable for the study was the teaching strategy of using manipulative and the dependent variable was students' understanding of trigonometry which was reflected in their performance. The students were divided into two groups, and one group was assigned as the experimental group, and the other as the control group. Each group included 70 students. At the beginning of the research study the experimental and control groups were pre-tested with on the prequisite areas of trigonometry. The test allowed the

reserchers to decide whether the groups were equivalent before the intervention programme was given out to the experimental group. The pre-test assessments assessed whether there was any academic performance discrepancy at the beginning of the analysis between the two group.

Statistical Package for Service Solutions (SPSS) software version 21.0 was used in the data analysis. Descriptives statistics such as means, standard deviations for pretest and posttest were calculated. In addition, percentages and frequencies were also used in the analysis of the pretest and posttest results. Mann-Whitney and Wilcoxon (MWW) test was also used to compare the differences between the marks obtained by the control and experimental groups.

Results

In this section, the results of experimental and control groups are described, analyzed and interpreted. It displays the collection of data from respondents through a TMT. The aim of this paper was to enhance students' understanding of basic trigonometry using concrete manipulatives. In order to answer the research question posed by this study, the pretest and posttest were conducted. Descriptive and inferential

statistics of the student's knowledge of the fundamental concept of trigonometry were measured before and after treatment using TMT on trigonometry. The sequence of results is consistent with the question and hypotheses presented in the paper.

Normality Test

The outcome from both groups was first tested for normality from the pre-test scores. It has been done to assess the distribution of the scores and to support the reserchers in the statistical methods to be used. There are many ways to find out if the data are normally distributed or not. The present study adopted the Kolmogorov-Smirnov and Shapiro-Wilk test to verify the normality of the distribution of the scores. In the Table 1 findings, the pre-test results were not normally distributed, as both the significant Kolmogorov-Smirnov values and the Shapiro-Wilk values were 0.00 lower than the 0.05 alpha value, thereby violating the normality assumption. Consequently, the statistical discrepancies of the control and the experimental groups on the pre-test scores were tested using a non-parametric test (Mann-Whitney and Wilcoxon). The normality of the pre-test scores is shown in Table 1.

Table 1 – Normality Test for Pre-Test Scores

Group of students	Kolmogorov-Smirnov ^a			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Control Group	0.267	70	0.000	0.760	70	0.000
Pre Test Score Experimental Group	0.192	70	0.000	0.913	70	0.000

a. Lilliefors Significance Correction

Table 2 showed that the experimental group were ranked above the control group in terms of the pretest scores with a mean rank of 104.70 and 98.30 respectively. Table 2 shows the rank of the independent variables used in the study to test the pretest scores.

Table 2 – Rank of Independent Variables for Pre-Test

	Group of students	N	Mean Rank	Sum of Ranks
Pre Test Score	Control Group	70	98.30	5541.00
	Experimental Group	70	104.70	7329.00
	Total	140		

The results of the pre-test scores using Mann Whitney and Wilcoxon test found no statistically significant difference ($p > 0.05, z = -10.146$) in the pretest scores between the control and experimental groups. Consequently, we retain the null hypothesis and conclude that, there is no statistically significant difference in the pretest scores between the control and experimental groups. We may thus infer that the mean was the same for the experimental and control groups. Thus at the inception of the study both groups were equal in academic ability. This allowed the researchers to introduce the use of manipulatives in teaching and learning of basic trigonometry to the experimental group. Table 3 shows the non parametric test statistics.

Table 3 – Non-Parametric Tests Statistics for Pre-Test

	Pre Test Score
Mann-Whitney U	56.000
Wilcoxon W	2541.000
Z	-10.146
Asymp. Sig. (2-tailed)	0.543
a. Grouping Variable: Group of students	

Post Test for the Control and Experimental Groups

In assessing the impact of the use of manipulatives on students’ performance, the research question: “What is the impact of the use of manipulatives on students’ performance in basic trigonometry?” was addressed. The post-testing was given to the control and experimental groups after the intervention. Both groups were taught basic trigonometry, but the use of manipulative was used to teach the experimental group while the control group were being taught by conventional teaching method. During this process, both groups discussed the same issues for a period of four weeks. The content coverage included introduction to trigonometry trigonometric ratios, sine and cosine rules and area of a triangle. Once the subject was taught extensively, both groups were tested concurrently and under the same conditions. The post-test trigonometry was developed to assess student conceptual knowledge and practical trigonometry comprehension. As measure of its trigonometric efficiency, the results obtained for the students during the posttests were used. The results are described and evaluated in stages. Tables 4 and 5 summarizes the scores obtained in the post-test by the students on the trigonometry test using the frequency and stem and leaf plots.

Table 4 – Post-Test Scores on a Stem and Leaf Plot for the Control Group (n = 70)

Frequency	Stem & Leaf
3	2. 000
8	3. 00000000
9	4. 000000000
16	5. 0000000000000000
23	6. 0000000000000000000000
4	7. 0000
3	8. 000
2	9. 00
2	Extremes (≥ 10.0)

Stem width: 1

Each leaf: 1 case(s)

Table 5 – Post-Test Scores on a Stem and Leaf Plot for the Experimental Group (n = 70)

Frequency	Stem & Leaf
3	5. 000
7	6. 0000000
6	7. 0000000
8	8. 00000000
21	9. 000000000000000000000000
25	10. 0000000000000000000000000000

Stem width: 1

Each leaf: 1 case(s)

The findings from Table 4 and Table 5 showed that students who had been subjected to the use of manipulatives performed better after the study than students who learned using conventional teaching strategies. In the experimental

Table 6 – Normality Test for Post-Test Scores

Group of students	Kolmogorov-Smirnov ^a			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Post Test Control Group	0.195	70	0.000	0.938	70	0.002
Score Experimental Group	0.262	70	0.000	0.826	70	0.000

a. Lilliefors Significance Correction

Table 7 shows, in terms of posttest results with a mean rank of 108,79 and 99,21, that the experimental group was ranked above the control group. Table 7 shows the rank of the independent variables used in the study to test the posttest scores.

Table 7 – Rank of Independent Variables for Post-Test

	Group of students	N	Mean Rank	Sum of Ranks
Post Test Score	Control Group	70	99.21	5954.50
	Experimental Group	70	108.79	7915.50
	Total	140		

group no students scored less than average score that is less than 50% in contrast to 20 students in the control group. The results were checked first in order to ascertain the normality of the distribution of the post-test scores. To verify if the scores were normally distributed, the Kolmogorov-Smirnov and the shapiro-wilk tests for normality were employed.

The results from Table 6 suggest that post-test scores were not normally distributed since both Kolmogorov-Smirnov and Shapiro-Wilk test had substantial values lower than an alpha value of 0.05, which thus violates the normality assumptions. A nonparametric test (Mann-Whitney and Wilcoxon) was therefore employed to check the statistical differences between the posttest nscores of the control and experimental groups. Table 6 displays results of the test for normality for the posttest scores.

The results of the post-test scores using Mann-Whitney and Wilcoxon test found a statistically significant difference ($p < 0.05, z = -8.360$) in the posttest scores between the control and experimental groups. Consequently, we reject the null hypothesis and conclude that, there is a statistically significant difference in the posttest scores between the control and experimental groups. We may thus infer that, following the intervention, the mean scores was not equal for the experimental and the control groups. These results suggest that the application of manipulatives would boost student academic performance. Furthermore, manipulations improve student performance and thus better results from the experimental group were obtained. The result also shows that the use of manipulatives has helped students to better understand the content of trigonometry. Table 8 shows the non parametric test statistics of post test scores.

Table 8 – Non-Parametric Tests Statistics for Post-Test

	Post Test Score
Mann-Whitney U	469.500
Wilcoxon W	2954.500
Z	-8.360
Asymp. Sig. (2-tailed)	0.000

a. Grouping Variable: Group of students

Discussion

The results of the study showed that students had little knowledge of their pre-tertiary education basic trigonometry. Few students were rated above the 5 (50%) average mark. The pretest shows that it was difficult for students to grasp the concept of the right-angled triangle, the concept of opposite, adjacent and hypotenuse and eventually the simple trigonometric ratios (sine, cosine and tangent) from a right-angled triangle. Consequently, it was not surprising students have been avoiding trigonometric questions

and wish not to meet them again at the tertiary level.

After the intervention, the students' performance in the experimental group improved dramatically. Unlike their counterpart in the experimental group, the students in the control group could not make more meaningful sense from the concept of trigonometry. This is evident in the post-test performance. Inadequate teaching aids, the absence of textbooks, inappropriate teaching methods and the non-use of manipulatives may be the cause of student inability to abstract and to conceptualize the idea of basic trigonometry. In other words, it can not be overlooked how critical student engagement, worksheets, teaching aids, motivational steps and the clear presentation of concepts is. The infrastructure, equipment and material facilities give the students the opportunity to acquire the knowledge they need (Sarfo, Eshun, Elen & Adentwi, 2014). In other words, due to the existence of manipulatives the output in the experimental group improved considerably. Although the control group's latest performance was higher but much less than the experimental group's results. This suggests that manipulatives are required, no matter how organized the lesson is and the teaching aids available. The results of the study support the assertion by Adipo (2015) that if a teacher uses appropriate manipulative to support his or her teaching, it enhances the students' innovative and creative thinking and consequently helps them to become more enthusiastic in learning the subject. The study reiterates the assertion by Cope (2015) and Kontas (2016) that, manipulatives do not only improve students' cognitive level but it goes extra mile to improve their psychomotor ability.

In mathematics, especially in basic trigonometry, the poor performance of students could be traced to failure of students to appreciate what they are taught.

Students must be placed in the center of each lesson. It should be possible for students to perform several tasks alone during mathematical instruction. This will contribute not only to improve mathematical performance but will support the interest of students in the subject.

Conclusion

The students in the experimental group performed better than their counterparts in the control group after the intervention, when the basic skills and concepts were properly manipulated. Thus, a proper use of manipulatives could help improve the performance of students.

Practical Implications

Teachers of mathematics must ensure that the use of manipulative materials are well combined with heuristic methods to allow students to be more interested in the instructional delivery and consequently contributing to the teaching and learning of trigonometry. Meeting the learning needs of students should move away from mechanical exercises into hands on activities with manipulatives which takes more time to use.

Teachers must be motivated to use their expertise in manipulatives through in-service training of practicing mathematics teachers by means of educational fora to learn and improve their awareness. These conferences would help teachers create a variety of education strategies that best meet their students' learning needs and enhance student participation in trigonometry teaching and learning. Efforts must be made to promote the understanding and assessment of the role of the teaching process by mathematics teachers through the use of manipulatives.

The Ministry of Education should integrate other educational tools in the form of information technology, such as computers, projectors, interactive whiteboards, etc. through the Free Education Policy of the

Government of Ghana. The students in the Colleges of Education are expected to fulfill their learning needs. In order to meet the current needs of the students, the use of manipulatives which helps ensure the standard and quantity of the latest educational services in our classrooms are recommended.

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