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Monitoring and Simulating Flow Rate in Groundwater Using Quadratic Shape Function Approach

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Abstract:

The research presented in this article is centered on monitoring and simulating flow rate in an unconfined formation. One dimensional Laplace equation of steady flow was used to develop flow rate model. The flow rate of Ede and Obagi Communities was also evaluated. The quadratic shape function of finite element approach was used to estimate flow rate at each node. An acceptable migration order of flow rate was observed as the considered flow rate initially moved from initial distance of zero to2.6km indicating fitness and usefulness of the quadratic shape function approximation concept in monitoring and simulating flow rate within the water bearing aquifer zone of the study area.

Key Words: Ede and Obagi Communities and aquifer zone.

Introduction:

The mathematical modeling of flow of groundwater through aquifer has become key interest to water resources engineers. This flow can be either surface runoff in rivers and streams, or subsurface runoff infiltrating rocks and soil or flow rate through aquifer. The amount of runoff reaching surface and groundwater can vary significantly, depending on rainfall, soil moisture, permeability, groundwater storage, evaporation, upstream use, and whether or not the ground is frozen. The movement of subsurface water is determined largely by the water gradient, type of substrate, and any barriers to flow [1-17]. Groundwater deep infiltration is a hydrologic process where water moves downward from surface water to groundwater. Recharge is the primary method through which water enters an aquifer. This process usually occurs in the vadose zone below plant roots and is often expressed as a flux to the water table surface. Recharge occurs both naturally (through the water cycle) and through anthropogenic processes (i.e., "artificial groundwater recharge"), where rainwater and or reclaimed water is routed to the subsurface. Flow within the soil body may take place under unsaturated conditions, but faster subsurface flow is associated with localized soil saturation [2-14].

Methodology:

Reconnaissance Assessment/ Wells Monitoring:

Selected boreholes in each of the communities of interest were monitored within five weeks to establish their water table height.

Mathematical Model of Aquifer Formulation Theory:

Geological investigations of the study area and hydraulic gradient observation based on water table and other environmental functional parameters observation in reconnaissance study of the studied vicinity an unconfined aquifer is considered. Figure 2 demonstrates the outline of groundwater flow between two rivers whose water levels are different. If the flow within water bearing aquifer zone is assumed to be one dimensional and steady state with a hydraulic conductivity (k). Then the Laplace equation is,

$$\frac{d^2h}{dx^2} = 0 \qquad (1)$$

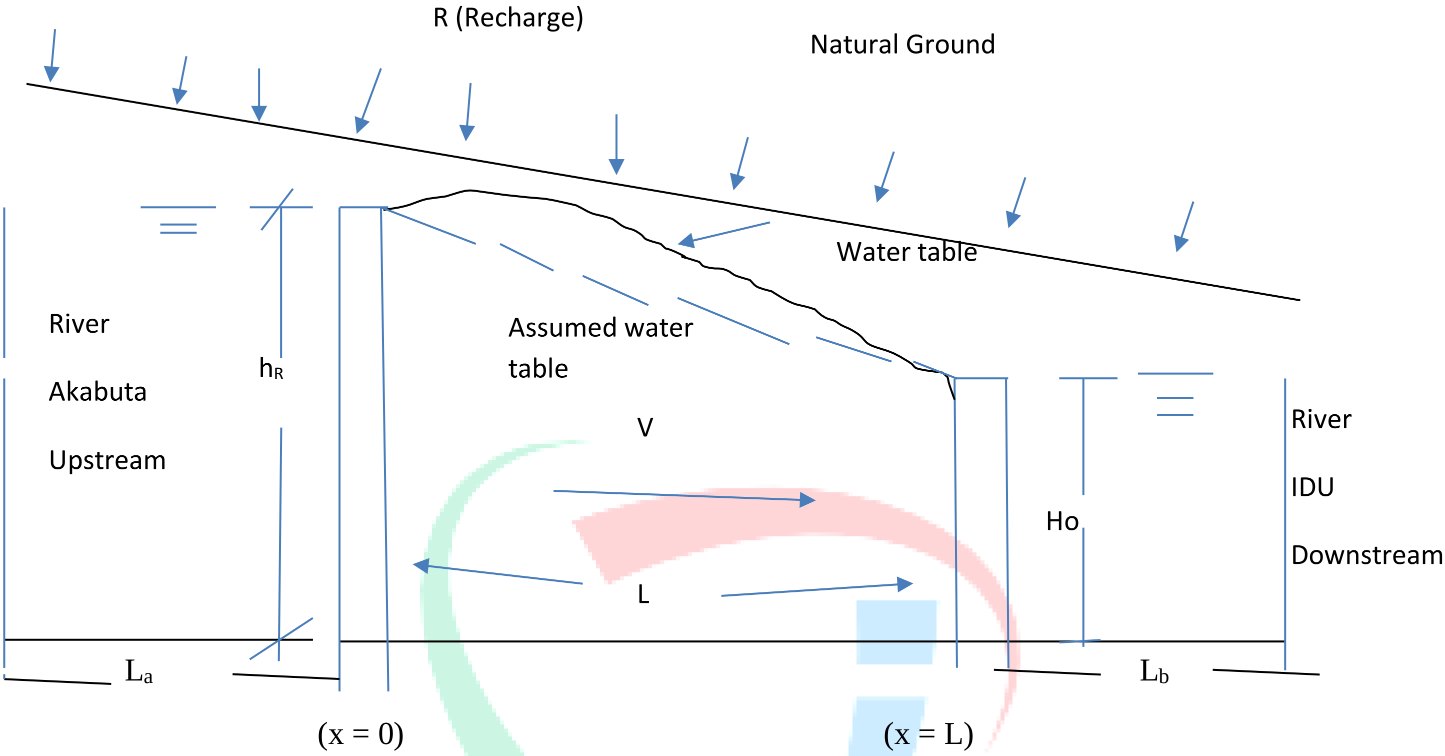

Figure 1. Unconfined aquifer formation

Integrating equation (1)

$$h^2 = ax + b \qquad (2)$$

Boundary condition 1: h = h_I at x = 0

Therefore, equation (2) becomes

$$b = h_I^2 \qquad (3)$$

Differentiating equation (2) with respect to x

$$\frac{2h\,dh}{dx} = a \qquad (4)$$

Application of Darcy's equation

$$q = -kh\frac{dh}{dx} \qquad (5)$$

Putting Equation (5) and Equation (3) into Equation (2) gives

$$h^2 = 2h\frac{dh}{dx}x + h_I^2$$

$$h^2 = h_I^2 - \frac{2qx}{k} \qquad (6)$$

Considering boundary condition 2: h = h_J at x = L

$$h_J^2 = h_I^2 - \frac{2qL}{k} \qquad (7)$$

Therefore,

$$q = \frac{k}{2L}(h_I^2 - h_J^2) \qquad (8)$$

Equation (7) yields

$$h^2 = h_I^2 - \frac{x}{l}(h_I^2 - h_J^2) \qquad (9)$$

Equation (9) gives the difference in height of the water table. Furthermore, the above equation holds only in the event of no recharge. Recharge is the proportion of rainfall that eventually finds its way in to the aquifer and raises the water level. If recharge is R, then

$$\frac{dq}{dx} = R \qquad (10)$$

Recalling Darcy's law

$$q = -kh\frac{dh}{dx}$$

Therefore,

$$\frac{dq}{dx} = -kh\frac{dh}{dx} = R \qquad (11)$$

Integrating equation (11) twice

$$h^2 = \frac{Rx^2}{k} + ax + b$$

As in the non- recharge case, boundary conditions are the same, giving

$$b = h_I^2$$

And $a = \frac{(h_J^2 - h_I^2)}{l} + \frac{Rl}{K}$

Putting and re-arranging, we arrive at

$$h = h_I^2 - \frac{x}{l}\left(h_I^2 - h_J^2\right) + \frac{Rx}{k}(l - x) \qquad (12)$$

This Equation (12) explained the profile of the water table line and parabolic. The water flow rate all the way through the water bearing aquifer is evaluated as given following.

Differentiating Equation (12) with respect x

$$2h\frac{dh}{dx} = \frac{h_I^2 - h_J^2}{l} + \frac{R}{k}(l - 2x) \qquad (13)$$

Solving Equation (14) becomes

$$\frac{-2q}{k} = \frac{h_I^2 - h_J^2}{l} + \frac{R}{k}(l - 2x)$$

$$q = \frac{k}{2L}\left(h_I^2 - h_J^2\right) - \frac{R}{2}(l - 2x) \qquad (14)$$

Equation (14) is equation for flow with the effect of recharge.

It is observed that flow rate varies with respect x.

Let recall Equation (11) and using a formulation model equation to simulate flow rate through aquifer.

$$\frac{dq}{dx} = R$$

Assimilation of Finite Element Method in Monitoring and Predicting Flow Rate:

Stage-1: Discretization and selection of Approximation function. Quadratic shape approximation is adopted in this study.

One – dimensional stretch was assumed and three elements with six nodes and the flow rate through the aquifer of each node were evaluated.

Stage – 2: deviation of element equations. Applying Galerkins Weighted Residuals Method GWRM to the governing one – flow rate equation (13) is expressed as:

$$\int_o^l E^T[\frac{dq}{dx} - R]\, dx = 0 \qquad (15)$$

Stage 3: Assembling of individual evaluated terms of Equation (15)

In Equation (11) one-dimensional stretch were assumed in this investigation in order to establish the assembling equation which will generate the flow rate at each node of intere

Results and Discussion:

Approach of Quadratic Shape Approximation Function of Finite Element:

$$C(x) = E_i^e Q_i + E_{i+1}^e Q_{i+1} + E_{i+2}^e Q_{i+2} = [E][Q] \qquad (16)$$

Where $E_e = (1 + \frac{x}{l})(1 - \frac{2x}{l})$ (17)

$$E_{i+1}^e = \frac{4X}{L}(1 - \frac{X}{L}) \qquad (18)$$

And

$$E_{i+2}^e = \frac{-X}{L}(1 - \frac{2X}{L}) \qquad (19)$$

Deviation of Element Equations

Individual evaluation of Equation (15)

Evaluating term 1 of Equation (15)

$$\int_o^l E^T \frac{\partial q}{\partial x} dx = \int_o^l E^T \frac{\partial}{\partial x}([E][Q])\, \partial x \qquad (20)$$

$$\int_o^l E^T E \frac{\partial Q}{\partial x} \partial x = \int_o^l \begin{vmatrix} (1+\frac{x}{L})(1 - \frac{2x}{L}) \\ \frac{4X}{L}(1 - \frac{X}{L}) \\ \frac{-X}{L}(1 - \frac{2X}{L}) \end{vmatrix} \left|[(1 + \frac{X}{L})(1 - \frac{2X}{L}) \quad \frac{4X}{L}(1 - \frac{X}{L}) \quad \frac{-X}{L}(1 - \frac{2X}{L})]\right| \begin{vmatrix} Q_1 \\ Q_2 \\ Q_3 \end{vmatrix} \partial x \qquad (21)$$

Solving Equation (21) gives:

$$= \frac{L}{30}\begin{vmatrix} 4 & 2 & -1 \\ 2 & 16 & 2 \\ -1 & 2 & 4 \end{vmatrix}\begin{vmatrix} Q_1 \\ Q_2 \\ Q_3 \end{vmatrix} \quad (22)$$

Evaluating term 2 of Equation (15)

$$\int_o^l E^T R \, \partial x = R \int_o^l \begin{vmatrix} (1+\frac{x}{L})(1-\frac{2x}{L}) \\ \frac{4X}{L}(1-\frac{X}{L}) \\ \frac{-X}{L}(1-\frac{2X}{L}) \end{vmatrix} \partial x \quad (23)$$

Solving Equation (24) gives

$$= \frac{RL}{6}\begin{vmatrix} -1 \\ 4 \\ -7 \end{vmatrix} \quad (24)$$

Assembling Equation (22) and (24) yields

$$\frac{L}{30}\begin{vmatrix} 4 & 2 & -1 \\ 2 & 16 & 2 \\ -1 & 2 & 4 \end{vmatrix}\begin{vmatrix} Q_1 \\ Q_2 \\ Q_3 \end{vmatrix} - \frac{RL}{6}\begin{vmatrix} -1 \\ 4 \\ -7 \end{vmatrix} = 0 \quad (25)$$

Monitoring flow rate at five different nodes and four elements by adopting the Galerkin's quadratic shape function of finite element method (FEM) gives,

$$\frac{L}{30}\begin{vmatrix} 4 & 2 & 1 & 0 & 0 \\ 2 & 16 & 2 & 0 & 0 \\ -1 & 2 & 8 & 2 & -1 \\ 0 & 0 & 2 & 16 & 2 \\ 0 & 0 & -1 & 2 & 4 \end{vmatrix}\begin{vmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{vmatrix} = \frac{RL}{6}\begin{vmatrix} 1 \\ 4 \\ -8 \\ 4 \\ -7 \end{vmatrix} \quad (26)$$

Apply model flow rate equation of unconfined aquifer Equation (15) gives

$$q_{E-O} = \frac{62.5}{2\,X3250}(18^2 - 16.9^2) - 5\,X\,10^{-5}(3250 - 2\,X3250) = 0.5\frac{m^3}{d}$$

q_{E-O} = Flow rate between Ede Community to Obagi Community ($\frac{m^3}{d}$ per m run)

Simulating $q_{BF} = 0.5\frac{m^3}{d}$ through the unconfined aquifer gives

***Table 1**. Flow rate evaluated by method of quadratic shape function of finite element method (FEM) against distance/ nodes*

Nodes	Distances (m)	FEM Flow Rate ($\frac{m^3}{d}$)
0	0	0.5
1	650	0.08
2	1300	0.09
3	1950	0.02
4	2600	0.03

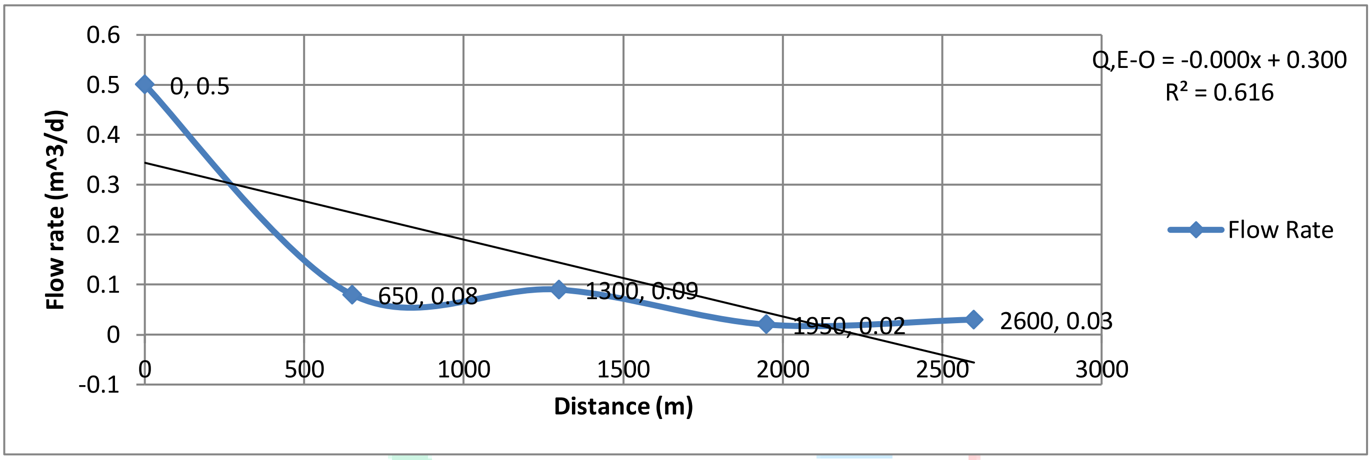

***Figure 2.** Plot of flow rate against distance*

Table 2 and Figure 2 illustrate the trend of behavior of flow rate through the aquifer upon distance of investigation. A flow rate of 0.5m^3/day per metre run were simulated using quadratic shape function of finite element method of Galerkin's to obtained flow rate at different nodes/ distance of interest, so considering inflow rate of 0.5m^3/day and outflow rate 0.03m^3/day was obtained by FEM. Fitting FEM flow rate into linear regression model established the equation of best fit as, Q, E-O = -0.000x + 0.300 and coefficient of determination R^2 = 0.616. A rise and fall in flow rate values were observed as shown in Figure 2 and this can be attributed to intrusion or seepage flow due to upstream river and rainfall percolation into the aquifer as of the time of assessment as a flow rate 0.5m^3/day is entering and 0.03m^3/day leaving at final distance of investigated length.

***Table 2**. Theoretical and finite element method (FEM) flow rate against distance*

Distances (m)	Theoretical Flow Rate ($\frac{m^3}{d}$)	FEM Flow Rate ($\frac{m^3}{d}$)
0	0.5	0.5
650	0.30	0.08
1300	0.30	0.09
1950	0.30	0.02
2600	0.30	0.03

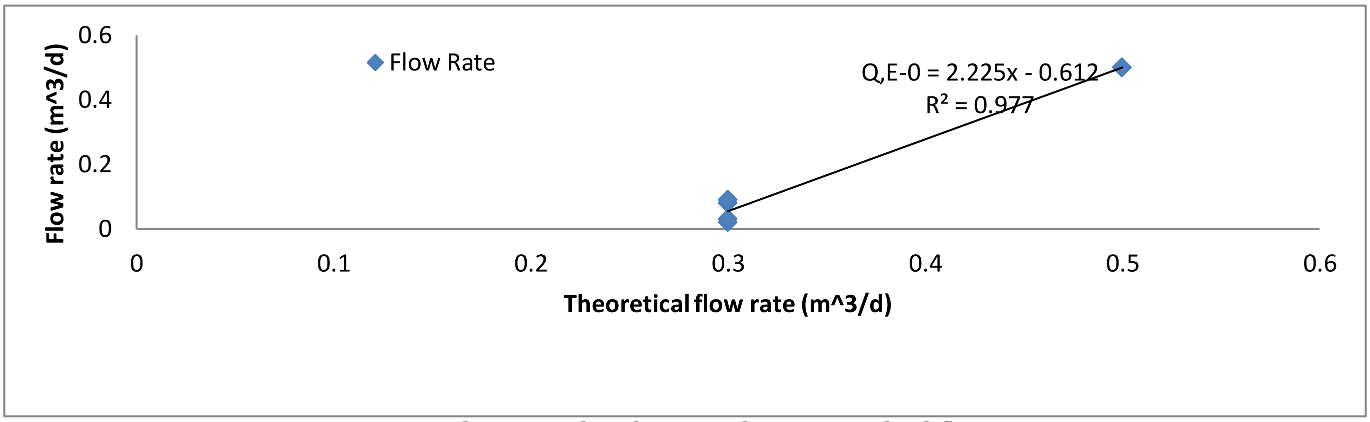

Figure 3. Theoretical and Finite element method flow rate

Table 3. *Validated and FEM flow rate against distance*

Distances (m)	Validated Flow Rate ($\frac{m^3}{d}$)	Flow Rate ($\frac{m^3}{d}$)
0	0.5	0.5
650	0.06	0.08
1300	0.06	0.09
1950	0.06	0.02
2600	0.06	0.03

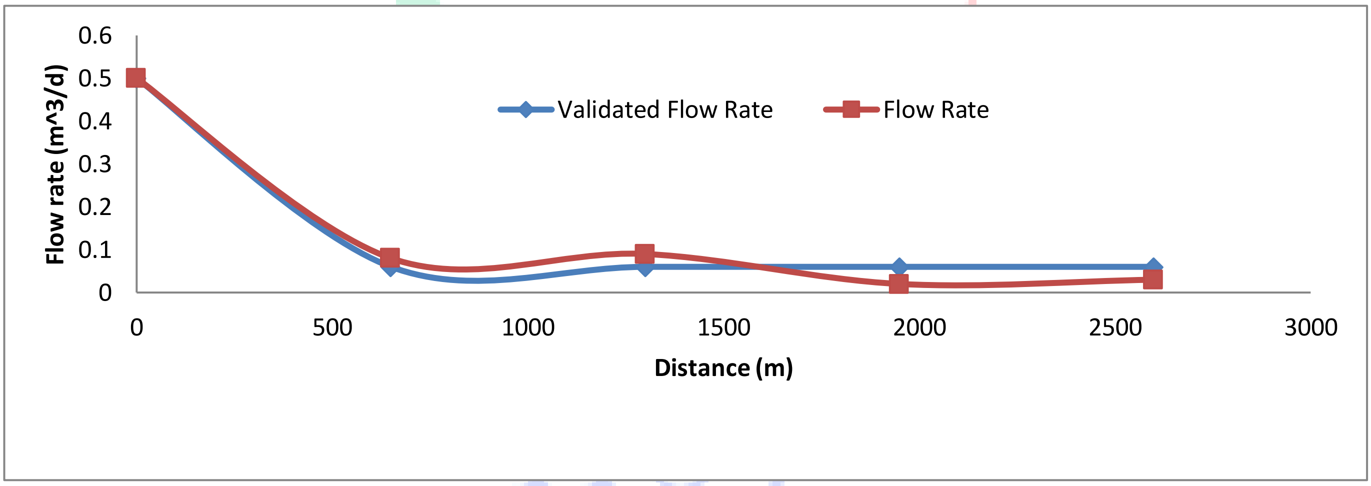

Figure 4. *Comparison between validated flow rate and finite element method Flow rate against distance*

Upon validation of flow rate model generated by studying the relationship between theoretical flow rate and FEM by mathematical tool of linear regression established a strong relationship as $R^2 = 0.999$ indicate that the linear regression model obtained is a valid model to be used in monitoring and predicting flow rate within Ede –Obagi Community water bearing zone. Figure 4 and Table 4 demonstrate the comparison between validated flow rate and finite element method against distance of investigation and acceptable pattern of flow rate in trend of migration flow as seen above was established.

Conclusion:

The assessment sucessfully studied the behavior of flow rate through the aquifer upon distance of investigation. A flow rate of 0.5m^3/day per metre run was simulated using quadratic shape function of finite element method of Galerkin's to obtain flow rate at different nodes/ distance of interest, so considering inflow rate of 0.5m^3/day and outflow rate 0.03m^3/day was obtained by FEM.

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