MHD axisymmetric flow and heat transfer of power law fluid over an unsteady porous stretching sheet

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Abstract: This paper deals with the boundary layer flow and heat transfer of power law fluid over an unsteady radially stretching sheet. By using the similarity transformation, the governing nonlinear boundary layer equations are transformed into nonlinear ordinary differential equation. These ordinary differential equations are then solved numerically by using BVP4C in MATLAB. The effects of different physical parameters like power law index n, unsteadiness parameter A, magnetic parameter M on the temperature profiles and velocity profiles as well as on the local skin friction coefficient and local Nusselt numbers are analyzed and discussed.

Key Words: Axisymetric flow, power law fluid, unsteady stretching.

Introduction

The investigation of flow over an extending sheet has pulled in the consideration of developing number of scientists in view of its different applications in various parts of engineering. These applications includes fiber undercoat, continuous casting, striating of foodstuff, expulsion of metal and polymer, fluidization of the gadget, flexible sheets drawing, substance treating devices etc. Transfer of heat over stretching surfaces are also important due to its wide application in fields of engineering and industrially. Application contains wire and fiber coating, extrusion of metal and polymer, reactor fluidization, food stuff processing, crystal growing, continuous casting, annealing, transpiration cooling, drawing of plastic sheets, exchangers and chemical processing equipment. The boundary layer flow and heat transfer because of stretching plate are of reasonable significance in fiber innovation and explosion process, and of theoretical enthusiasm as well. The construction of polymer sheets and plastic films depend on this technology. There are number of precedents which incorporate the cooling of a vast metallic plate in cooling bath, the boundary layer along material conveyers, the extrusion of plastic sheet, paper production, glass blowing, and polymer expulsion.

The investigation of non-Newtonian liquids has pulled in much regard for the scientists in light of their reasonable applications, for example, polymer drilling mudd, grease process, food products, circulation of blood, and so forth. Various modernly significant liquids including liquid plastics, polymers, pulps, nourishment and non-renewable fuel sources, which may immerse in

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underground beds, show non-Newtonian conduct. Different constitutive conditions have been proposed, for example, Ostwald de Waele model , Binghan model, Casson model, Maxwell model, Jeffrey model, Olroyd-B model, etc, to clarify the conduct of the non- Newtonian liquids. Among these models, the power law model is the most broadly utilized in numerous fields of utilizations as a result of its straightforwardness. The power law exhibit has been comprehensively used in engineering process. Cortell [9] explored the MHD non-Newtonian power-law fluid flow and heat transfer over stretching sheet. Few researcher [10−12] examined the heat exchange of power law fluid of various physical model. Sakiadis [13] initiated the study of boundary layer flow on a moving continuous solid surface. Shardian [14] examined similarity solutions for unsteady boundary layer flow and heat transfer due to stretching sheet. Ibrahim and Shanker [15] studied the unsteady MHD boundary layer flow and heat transfer due to stretching sheet in the presence of heat source.

Wang [1] firstly examined the flow problem inside a thin film of Newtonian liquid over an unsteady stretching sheet. In [2] Crane studied the boundary flow of Newtonian fluid caused by stretching of an elastic flat sheet that moves in its own plane with a velocity varying linearly with a distance from a fixed point because of the application of uniform stress .Then in [3] Gupta added heat and mass transfer on a stretching sheet. Pop et al. [4] studied the flow and heat transfer in a nano-liquid film over an unsteady stretching surface. Lin et al. [5] discussed Magnetohydrodynamic thin film and transfer of heat of power law fluid over an unsteady stretching sheet with variable thermal conductivity. Khan at al. [6] discussed the behavior of flow of and exchange of heat of a second-grade liquid through a permeable medium past an extending sheet. Aziz et al. [7] examined the thin film flow and heat transfer over on an unsteady stretching sheet. Mahmoud [8] explored the effects on the flow and heat transfer in a liquid film on an unsteady stretching sheet. Prashant et al. [16] examined the thin film flow and heat transfer over an unsteady stretching sheet with thermal radiation in the existence of thermal radiation. liu et al. [17] examined by homotopy perturbation method to show the thin film flow and heat transfer over an unsteady stretching sheet with thermal radiation.

As it is already mentioned that most of the published work related to flow and heat transfer are available about power law fluid in Cartesian systems, where very less literature is available in the non-Newtonian fluids in Cylindrical system. The advancement of research in the field of radial stretching surfaces and its application in industry and technology is the need of the existing era. Therefore, the determination of existing research is based on the study of Power law fluid flow and heat transfer over an unsteady stretching surface in cylindrical system.

In this article we examined the flow and heat transfer of power law fluid over an unsteady radially stretching sheet. The basic equations are converted into non-linear ordinary differential equation with the help of similarity transformation. Numerical solutions are obtained by BVP4C in MATLAB. The resultant differential parameter are also discussed and presented graphically.

Mathematical formulation

We assume two dimensional flow of power law fluid over an unsteady radially stretching sheet. A uniform magnetic field $B(r,t) = B_0/(1-\alpha t)^{\frac{1}{2}}$ is applied perpendicular to the sheet. The cylindrical coordinates (r, θ, z) is considered for mathematical description. All physical

quantities are not depend upon θ because of rotational symmetry of flow. u denotes the component of velocity along r direction and w denotes the component of velocity along z direction. We consider the two dimensional velocity field $V = (u, 0, w)$. The governing nonlinear boundary layer equations for the flow and heat transfer are as follows,

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}) = -\frac{K}{\rho}\frac{\partial}{\partial z}\left(-\frac{\partial u}{\partial z}\right)^n - \frac{\sigma B_0^2 u}{\rho}, \tag{2}$$

$$\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = \alpha\frac{\partial^2 T}{\partial z^2}, \tag{3}$$

where $\alpha = \frac{k}{\rho c_p}$ is thermal diffusivity, $v = \frac{K}{\rho}$ is the kinematic viscosity, n is the flow behavior index. The boundary conditions for velocity and temperature fields are given by

$$u = u_w(r,t),\; w = -f_w(r,t),\; T = T_w = T_\infty + \frac{br}{1-\alpha t} \text{ at } z = 0, \tag{4}$$

$$u \to 0, T \to T_\infty \text{ as } z \to \infty, \tag{5}$$

$f_w(r,t)$ represents the porosity of the sheet, $u_w(r,t) = \frac{cr}{1-\alpha t}$ is the stretching surface velocity where $t < 1\backslash\alpha$. T_w and T_∞ denotes the surface temperature and ambient fluid temperature respectively. To solve the governing equations into a set of ordinary differential equations we introduce the following dimensionless transformation

$$\psi(r,z) = -r^2 U R_e^{\frac{-1}{n+1}} f(\eta),\; \eta = \frac{z}{r}\mathrm{Re}^{\frac{1}{n+1}},\; \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \tag{6}$$

In above transformation, η is the independent variable, θ is the dimensionless temperature The stream function ψ is defined as

$$u = \frac{1}{r}\frac{\partial\psi}{\partial r},\; w = -\frac{1}{r}\frac{\partial\psi}{\partial z}, \tag{7}$$

Re is the local Reynolds number defined as

$$\mathrm{Re} = \frac{\rho}{k}r^n U^{2-n}, \tag{8}$$

hence the velocity component can be written as

$$u = \frac{cr}{1-\alpha t}f',\; w = -\frac{cr}{1-\alpha t}\mathrm{Re}^{\frac{-1}{n+1}}\left[\frac{3n+1}{n+1}f(\eta) + \frac{1-n}{n+1}\eta f'\right]. \tag{9}$$

By using the above transformations the equation of continuity is identically satisfied and momentum Eq. (2) and energy Eq. (3) can be written as

$$A\left(f' + \frac{2-n}{1+n}\eta f'\right) - \frac{3n+1}{n+1}ff'' + f'^{2} = n(-f'')^{n-1}f''' - Mf', \tag{10}$$

$$\theta'' = \Pr A\left(\theta + \frac{2-n}{n+1}\eta\theta'\right) - \Pr\left(\frac{3n+1}{n+1}f\theta' - f'\theta\right), \tag{11}$$

where $M = \frac{\sigma B_0^2}{\rho c}$ the magnetic parameter, $A = \frac{\alpha}{c}$ the unsteadiness parameter, $\Pr = \frac{r u_w}{\alpha}\mathrm{Re}^{\frac{-2}{n+1}}$ is the pr and tl number. After using similarity transformation the boundary conditions $(4-5)$ are converted into following equations

$$f(0) = S,\ f'(0) = 1,\ \theta(0) = 1,\ \theta(\infty) = 0,\ f'(\infty) = 0, \tag{12}$$

where $S = \frac{3n+1}{n+1}\frac{\mathrm{Re}^{\frac{1}{n+1}}}{u_w}f_w$ is the mass transfer parameter. $S < 0$ for mass injection and $S > 0$ for mass suction. Physical quantities of our interest are local Nusselt number and local skin friction coefficient defined as

$$C_f = \frac{T_w}{\frac{1}{2}\rho u_w^2},\ N_u = \frac{r q_w}{k(T_w - T\infty)}, \tag{13}$$

Such as

$$T_w = \left(k\left|\frac{\partial u}{\partial z}\right|^{n-1}\right)\frac{\partial u}{\partial z}|_{z=0}. \tag{14}$$

and

$$q_w = -k\left(\frac{\partial T}{\partial z}\right)|_{z=0}. \tag{15}$$

After putting the values of T_w and q_w in Equation (13) we get

$$\frac{1}{2}\mathrm{Re}^{\frac{1}{n+1}}C_f = -[-f''(0)]^n,\ \mathrm{Re}^{\frac{-1}{n+1}}N_u = -\theta'(0). \tag{16}$$

Solutions of the problem
By using similarity transformation the governing nonlinear partial differential equation are reduced to a system of coupled nonlinear ordinary differential equations which are solved numerically by using BVP4C in MATLAB. For this we transform the boundary value problem into an initial value problem by introducing new set of variable which are given as

$f = y(1)$

$f' = y(2)$

$f'' = y(3) \tag{17}$

$\theta = y(4)$

$\theta' = y(5) \tag{18}$

Equation $(10-11)$ becomes

$$f''' = A(y(2) + ((2-n)/(n+1))\eta y(2) - ((3n+1)/(n+1))y(1)y(3) + (y(2)\wedge 2) +$$
$$My(2)/(n(-y(3)\wedge(n-1)) \qquad (19)$$

$$\theta'' = \Pr(A(y(4) + ((2-n)/(n+1))\eta y(5)) - ((3n+1)/(n+1))y(1)y(5) + y(2)y(4). \qquad (20)$$

Corresponding boundary conditions are

$y(1)[0] = S, y(2)[0] = 1, y(2)[\infty] = 0, y(4)[0] = 1, y(4)[\infty] = 0.$

And then it is unrevealed for several values of the involved parameters.

Numerical results and discussion

In order to examine the results, numerical computation has been carried out for different values of magnetic parameter M, generalized pr and tl number $\Pr$, unsteadiness parameter A, Injection parameter $(S < 0)$, Suction parameter $(S > 0)$. The modified differential equations associated with the boundary conditions are solved numerically by using BVP4C in MATLB. The impact of power law index n on temperature and velocity profiles is described in Figs. $(2-7)$ for shear thinning $(n < 1)$, Newtonian $(n = 1)$, shear thickening $(n > 1)$ fluids, respectively.

Fig 2 $(a-c)$ shows the effects of magnetic parameter M on the velocity field $f'(\eta)$. It is observed that with the increase in the value of magnetic parameter M, leads to decrease in velocity profile $f'(\eta)$ which indicates that rate of transport is noticeably diminish with increase in magnetic parameter. It shows that transverse magnetic field resist the transport phenomena because of variation of Lorentz force which produces opposition to transport phenomena. Further it is observed that effect of magnetic parameter M is less dominating for higher values of power law index n. Fig. 3 $(a-c)$ allocate the impact of unsteadiness parameter A on the velocity profile $f'(\eta)$. As unsteadiness parameter A increases, the velocity profile decreases because momentum boundary layer thickness decreases with increase of A. The flow of fluid arises due to stretching of sheet so when η increases velocity profile decreases and it satisfies boundary condition when $\eta = \infty$. Fig. 4 $(a-c)$ demonstrate the effect of suction parameter $(S > 0)$ on the velocity profile $f'(\eta)$. It seems that by increasing the value of S proceed to increase adherence of the fluid to the walls which in turn slowdown the flow, as a result velocity profile $f'(\eta)$ decreases. Fig. 5 $(a-c)$ describes the effect of injection parameter $(S < 0)$ on the velocity profile $f'(\eta)$. It is observed that by decreasing the value of S increases the velocity profile $f'(\eta)$. Fig. 6 $(a-c)$ demonstrate the effects of reformed pr and tl number $\Pr$ on the temperature profile $\theta(\eta)$. With the increase of $\Pr$ temperature decreases as the thermal boundary layer thickness decreases because of increase in $\Pr$. The temperature die out at the free surface for higher values of $\Pr$. This is analogues to the condition in both steady and unsteady state aerodynamic boundary layer in an infinite medium for greater values of pr and tl number. That is, thermal boundary layer thickness lessens for greater values of $\Pr$. In Fig. 7 $(a-c)$ the influence of unsteadiness parameter A on the temperature profile $\theta(\eta)$ is

discussed. The temperature profile $\theta(\eta)$ decreases with the increase of unsteadiness parameter A. As fluid flow is only due to stretching of sheet and stretching sheet temperature is greater than the ambient fluid temperature i.e $T_w > T_\infty$. Due to this reason temperature decreases when η increases and thermal boundary layer thickness decreases when unsteadiness parameter A increases.

Fig. 8 $(a-b)$ reveals the influence of power law index n on the velocity profile $f^{'}(\eta)$ and temperature profile $\theta(\eta)$. The Fig. 8(a) and 8(b) clearly shows that by increasing value of power law index tends to decrease in velocity and temperature profiles. This is a well-known fact that for larger values of n implies drag and hence decrease in velocity and temperature.

Table (1) and Table (2) shows that C_f and N_u increases with increase in the value of parameters for various values of power law index n respectively.
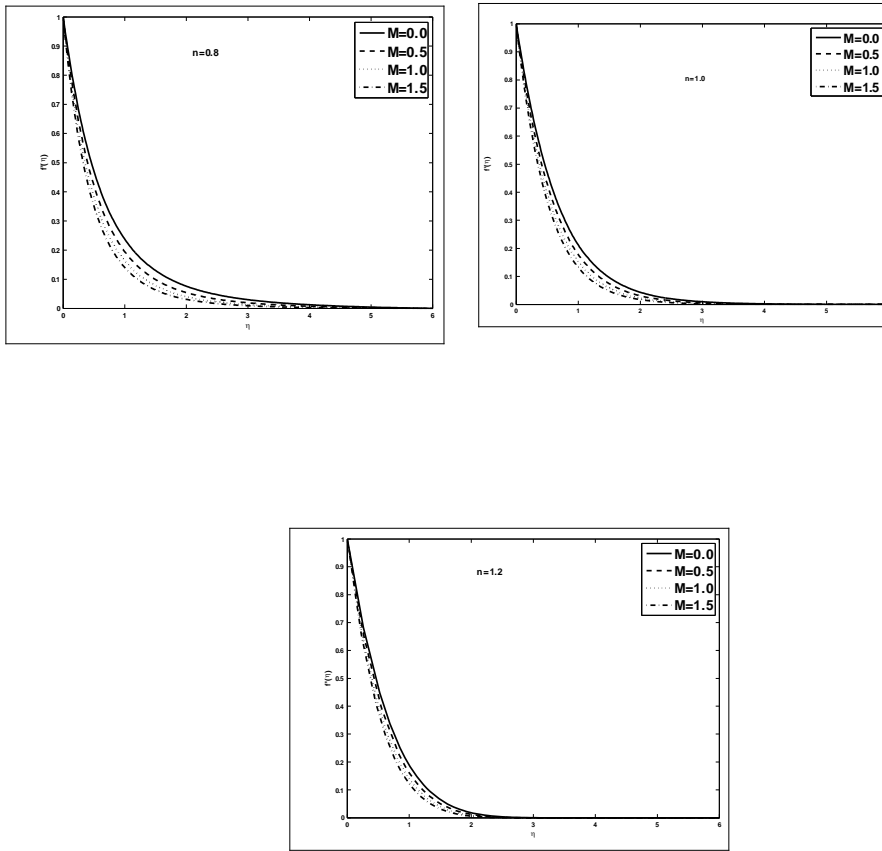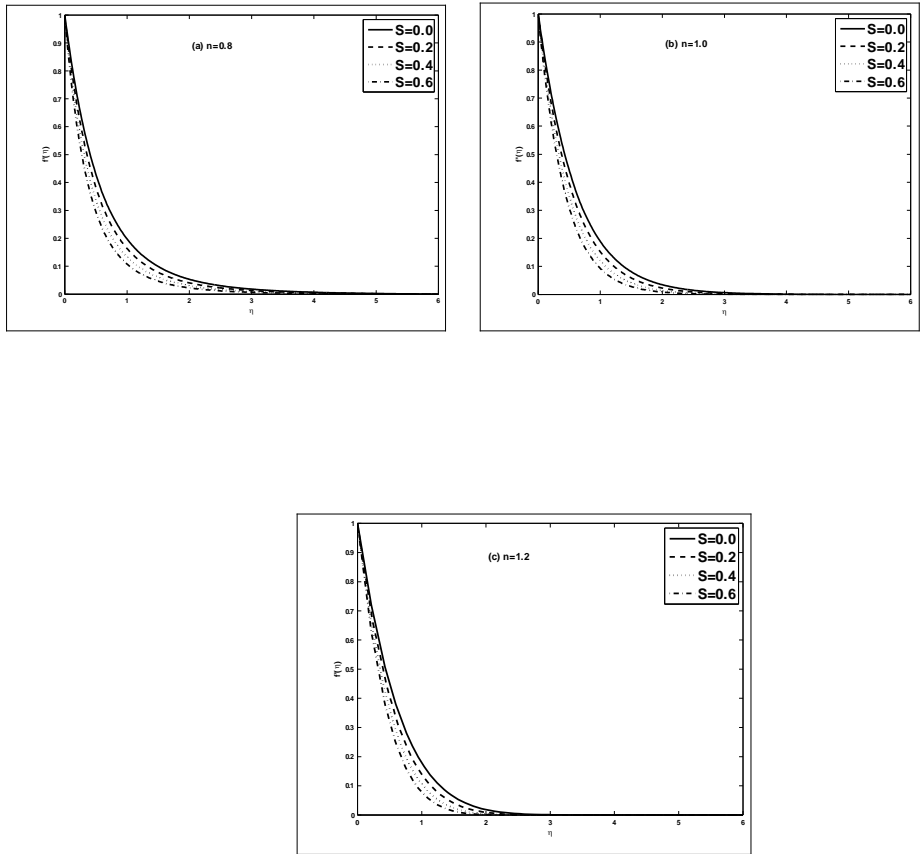
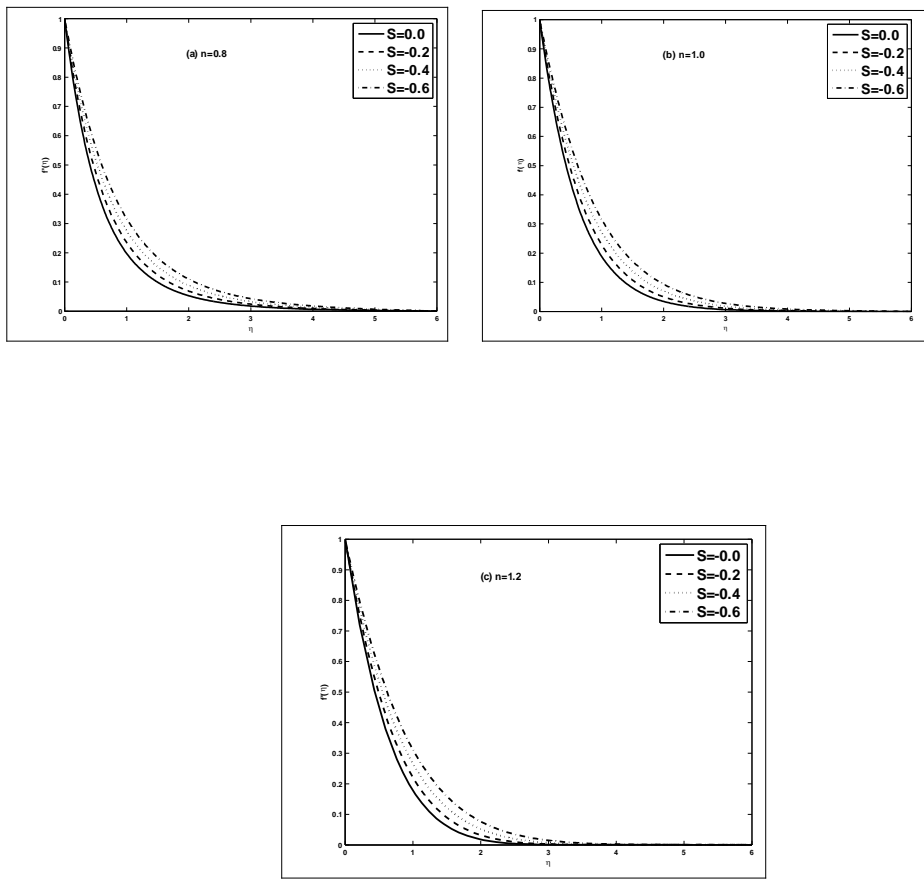**Figure : 2** Influence of magnetic parameter M on the velocity profile for S=A=0.2, Pr=0.7.
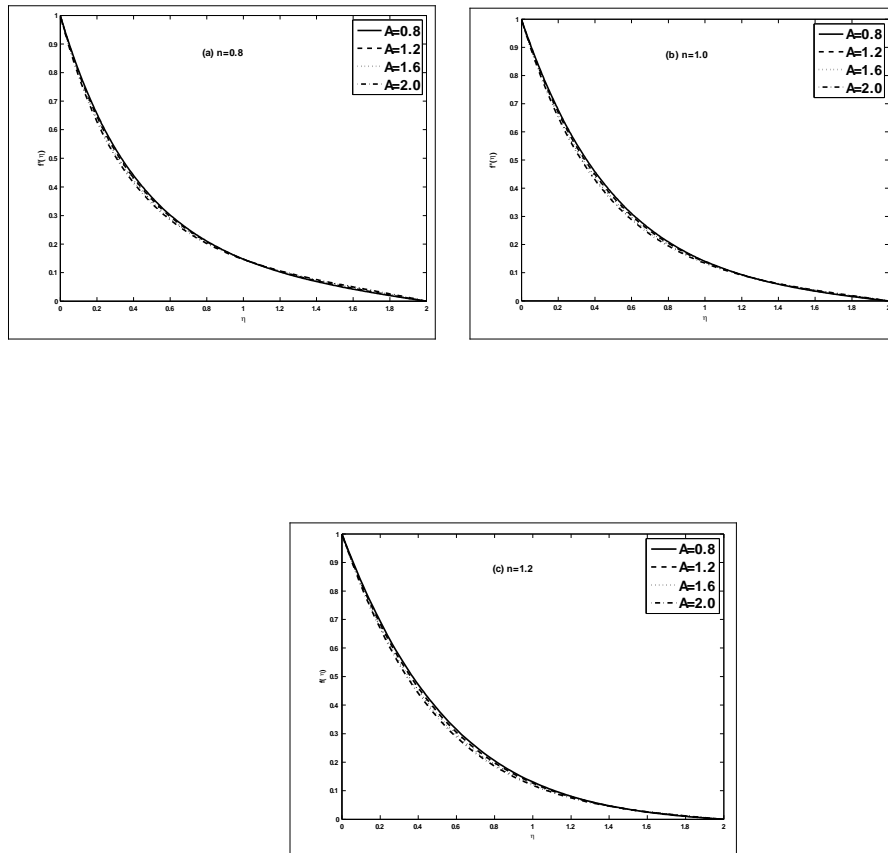
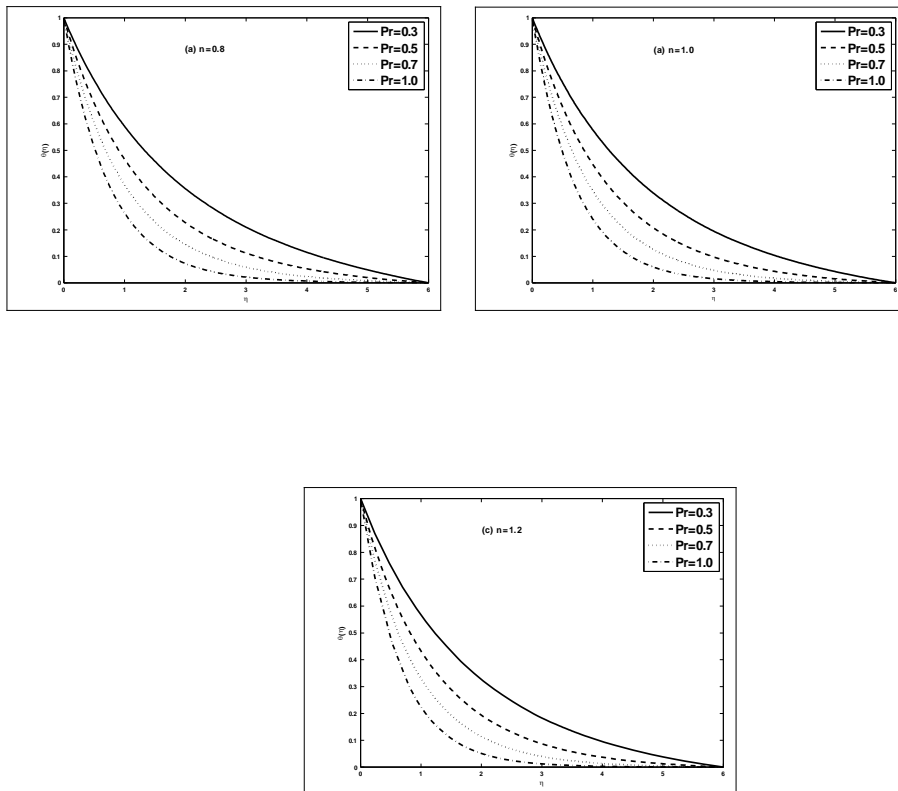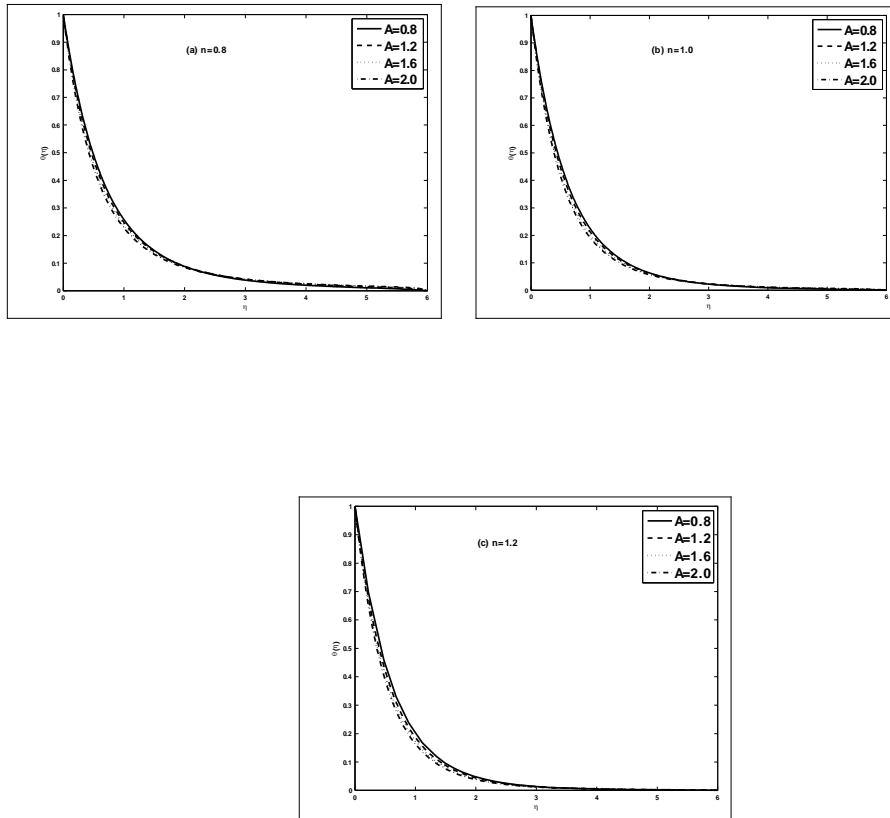**Figure: 3** Influence of unsteadiness parameter A on the velocity profile f $'(\eta)$ with S=0.2, M=1

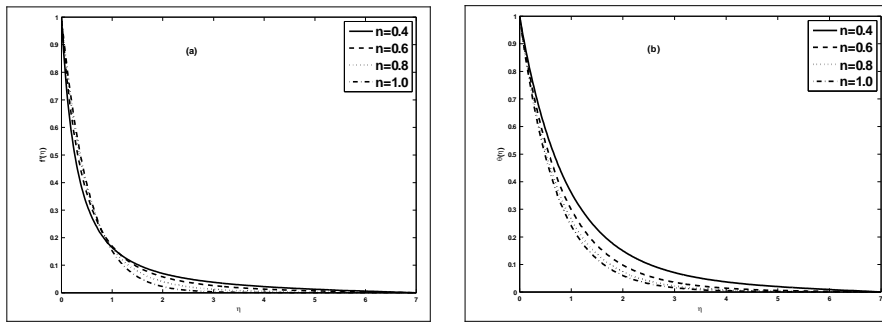**Figure: 4** Influence of Suction parameter S>0 on the velocity profile with A=0.2, M=1

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Figure: 5 Influence of Injection parameter S<0 on the velocity profile $f'(\eta)$ with A=0.2, M=1

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Figure : 6 Influence of Prandtl number Pr on the temperature profile when M=1, A=0.2=S

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Figure: 7 Influence of unsteadiness parameter A on the temperature profiles $\theta(\eta)$ with M=1=Pr, S=0.2.

Figure: 8 Influence of power law index n on (a) velocity profile $f^{'}(\eta)$ (b) temperature profiles when M=Pr=1.0, S=A=0.2.

Table1 Numerical values of the skin friction coefficient $\frac{1}{2}\mathrm{Re}^{\frac{1}{n+1}}C_f$ for various values of physical parameters values of the skin friction coefficient $\frac{1}{2}\mathrm{Re}^{\frac{1}{n+1}}C_f$ for various values of physical parameters.

A	M	S	n=0.8	n=1.0	n=1.2
0.0	1.0	0.2	1.71249	1.75664	1.79564
0.2			1.74725	1.79894	1.84506
0.5			1.79913	1.86166	1.91797
0.2	0.0		1.41927	1.45257	1.48747
	0.5		1.59528	1.63582	1.67447
	1.0		1.74725	1.79894	1.84506
		0.0	1.55566	1.58058	1.60183
		0.2	1.74725	1.79894	1.84506
		0.5	2.07534	2.16993	2.25518

Table 2 Numerical values of the local Nusselt number $\mathrm{Re}^{\frac{1}{n+1}}N_u$ for various values of physical parameters

A	M	S	Pr	n=0.8	n=1.0	n=1.2
0.0	1.0	0.2	0.7	0.97874	1.01406	1.04211
0.2				1.02321	1.06909	1.10577
0.5				1.09447	1.15044	1.19517
0.2	0.0			1.09569	1.12810	1.53662
	0.5			1.05591	1.09611	1.12802
	1.0			1.02321	1.06909	1.10577
		0.0		0.87953	0.911083	0.93674
		0.2		1.02321	1.06909	1.10576
		0.5		1.27398	1.34315	1.39844
		0.2	0.5	0.80067	0.83278	0.80067
			0.7	1.02321	1.06909	1.02321
			1.0	1.18316	1.38900	1.55642

Concluding remarks

We treated the unsteady boundary layer equations of power law fluid over an unsteady stretching sheet. After using the similarity transformation we reduce the partial differential equations into ordinary differential equations and then solved numerically by using BVP4C in MATLAB. We concluded that velocity was decreased with an increase of unsteadiness parameter A, suction parameter (S>0) and magnetic parameter M. The effects of pr and tl number on temperature profile was to decrease the temperature. Velocity increased with an increase of injunction parameter (S<0).

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