

Study of Heavy Meson Masses Using Spin-Dependent Potentials

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ABSTRACT

Heavy mesons are among the most useful hadronic systems for studying the interaction between quarks in the nonperturbative region of quantum chromodynamics. In particular, heavy quarkonium systems such as charmonium $\bar{c}c$ and bottomonium $\bar{b}b$ provide a comparatively clean environment for testing phenomenological quark–antiquark potentials, because the heavy constituent quarks move more slowly than light quarks and can often be treated within nonrelativistic or semirelativistic frameworks. This article investigates the mass spectra of heavy mesons using spin-dependent potentials. The meson is modeled as a two-body bound state composed of a quark and an antiquark. The total mass is obtained from the constituent quark masses together with the binding energy generated by the interaction potential. Several phenomenological potentials are discussed, including the Martin potential, Song–Lin potential, Turin potential, logarithmic potential, and Cornell-type quark–antiquark potential. The spin-independent part of the potential describes short-range Coulomb-like attraction and long-range confinement, while the spin-dependent part accounts for hyperfine, spin–orbit, and tensor splittings. These corrections are essential for distinguishing pseudoscalar and vector states such as η_c and ψ , as well as η_b and Y . The article shows that spin-dependent potential models remain valuable tools for describing heavy meson spectroscopy, especially when their predictions are compared with experimental data and established theoretical studies.

Keywords: heavy mesons, heavy quarkonium, charmonium, bottomonium, spin-dependent potential, quark–antiquark model, hyperfine splitting, meson spectroscopy.

1. INTRODUCTION

The classification and interpretation of hadrons have been central problems in particle physics since the rapid growth of experimentally observed strongly interacting particles during the twentieth century. Before the quark model, the increasing number of hadrons made it difficult to regard all of them as elementary particles. The quark model provided a powerful organizing principle by proposing that mesons are composed of quark–antiquark pairs ($q\bar{q}$), whereas baryons are composed of three quarks (qqq). This classification made it possible to interpret hadron multiplets, charge assignments, spin states, flavor structure, and many observed mass patterns within a unified framework. Standard particle-physics texts such as Griffiths [1] present this quark-model structure as a basic foundation for understanding modern hadron spectroscopy. The provided thesis also introduces mesons in Chapter 5 through the quark antiquark picture and then uses this structure as the starting point for calculating heavy meson masses with spin-dependent potentials.

Among hadronic systems, heavy mesons occupy a particularly important position. A heavy meson contains at least one heavy quark, usually charm (c) or bottom (b). When the meson is formed from a heavy quark and its corresponding antiquark, the system is called heavy quarkonium. The two most important heavy quarkonium families are charmonium $\bar{c}c$ and bottomonium $\bar{b}b$. These systems are especially useful for theoretical study because the relatively large charm- and bottom-quark masses reduce the importance of some relativistic effects compared with light-quark systems. As a result, the motion of the heavy quark and antiquark can often be approximated by nonrelativistic or semirelativistic bound-state equations. This makes heavy quarkonium a natural testing ground for potential models of the strong interaction.

Potential models are valuable because they provide a practical way to describe the complicated quark–antiquark interaction in the nonperturbative region of quantum chromodynamics. In these models, the interaction is usually represented by an effective

potential that contains a short-range attractive part and a long-range confining part. The short-range term is commonly associated with one-gluon exchange and has a Coulomb-like form, while the long-range term represents confinement and prevents quarks from appearing as isolated free particles. This physical structure is reflected in Cornell-type potentials, which have been widely used in quarkonium spectroscopy. The thesis bibliography includes several foundational works on quark–antiquark potentials and heavy-quarkonium spectra, including Eichten et al. [2] on charmonium, Ebert et al. [3] on heavy-quarkonium mass spectra, and Rai et al. [4] on QQ^- mesons in nonrelativistic QCD formalism.

Historically, potential models became especially influential after their successful application to charmonium. Eichten et al. [2] showed that a potential-model description could reproduce many features of the charmonium spectrum and provide meaningful comparison with experimental measurements. Similar ideas were then applied to bottomonium, where the larger bottom-quark mass makes the nonrelativistic approximation even more reliable. Lichtenberg et al. [5], for example, tested static quark–antiquark potentials using bottomonium, while Ebert et al. [3] developed a quark–antiquark potential that included retardation and radiative contributions for heavy-quarkonium mass spectra. These studies show that potential models are not merely mathematical fitting tools; they encode essential physical features of QCD, including short-distance attraction and long-distance confinement.

Several phenomenological potentials have been proposed to describe heavy meson spectra. The Martin [6] potential was introduced to provide a simultaneous fit to several quark–antiquark systems, including (bb^-) , (cc^-) , (ss^-) , and (cs^-) spectra. The Song and Lin [7] potential offers another form for modeling the radial dependence of the quark–antiquark interaction. The Turin potential, associated with Lichtenberg and collaborators, was used in studies of static quark–antiquark potentials and bottomonium spectroscopy [5]. The logarithmic potential is also important because it can describe certain regularities in quarkonium level spacings, as discussed by Quigg and Rosner [8]. These potentials are directly relevant to the provided thesis, because Chapter 5 is organized around the Martin, Song–Lin, Turin, logarithmic, and quark–antiquark potentials for heavy meson mass calculations.

Although the central, spin-independent part of the potential determines the main binding structure of a meson, it is not sufficient to reproduce the full observed mass spectrum. Mesons with the same quark content and the same radial and orbital quantum numbers can still have different masses because their spin configurations are different. A quark and an antiquark each have spin $(1/2)$, so their total spin can combine either into a spin-singlet state ($(S=0)$) or a spin-triplet state ($(S=1)$). This distinction is physically

important. For example, (η_c) and (J/ψ) are both charmonium states, but (η_c) is a pseudoscalar spin-singlet state, whereas (J/ψ) is a vector spin-triplet state. Their mass difference is a hyperfine splitting caused primarily by the spin–spin part of the interaction. The same idea applies to bottomonium states such as (η_b) and (Υ) .

The role of spin-dependent interactions in hadron spectroscopy was emphasized in early quark-model work by De Rújula et al. [9], who connected hadron mass splittings with spin-dependent forces. Later studies developed this idea in more detailed forms. Ebert et al. [10], for example, studied hyperfine splitting and leptonic decay rates in heavy quarkonia, while Ikhdaïr and Sever [11] investigated the (B_c) meson spectrum and hyperfine splittings using the shifted large- (N) expansion technique. These works are included in the source file’s bibliography and support the use of spin-dependent potentials as a central tool in heavy meson spectroscopy.

The inclusion of spin-dependent terms is especially important because hyperfine splitting decreases as the quark mass increases. In many potential models, the spin–spin interaction is inversely proportional to the product of the constituent quark masses, $(\frac{1}{m_1 m_2})$.

Therefore, the splitting between (η_b) and (Y) is expected to be smaller than the splitting between (η_c) and (J/ψ) . This mass dependence provides an important test of the potential model: a successful model should not only reproduce the absolute masses of heavy mesons, but also describe the pattern of spin splittings across different quarkonium families.

The present article therefore focuses on the calculation of heavy meson masses using spin-dependent quark–antiquark potentials. Building on the framework presented in Chapter 5 of the source thesis, the study considers heavy mesons as two-body bound systems and discusses how different phenomenological potentials can be used to calculate their mass spectra. Particular attention is given to the role of spin-dependent corrections in separating pseudoscalar and vector states. The goal is to show how potential models, despite their phenomenological nature, provide a useful and physically meaningful framework for understanding the spectroscopy of heavy mesons such as charmonium and bottomonium.

2. MATERIALS AND METHODS

2.1 Mesons in the Quark–Antiquark Model

In the constituent quark model, a meson is represented as

$$M = qq^-$$

where (q) denotes a quark and $(\bar{q})$ denotes an antiquark. This structure gives mesons zero baryon number, because the baryon number of a quark is $(+1/3)$, while the baryon number of an antiquark is $(-1/3)$. Therefore,

$$B(qq^-) = \frac{1}{3} - \frac{1}{3} = 0$$

The quark–antiquark model successfully explains the basic organization of mesons, including their charge, spin, parity, and flavor properties. In this framework, different meson families arise from different quark–antiquark combinations. Light mesons may contain (u), (d), or (s) quarks, while heavy mesons contain charm or bottom quarks.

A quark and an antiquark each have spin (1/2). Their total spin can combine in two possible ways:

$$S = 0$$

or

$$S = 1$$

When the spins are anti-parallel, the total spin is ($S=0$), and the state is a pseudoscalar meson. When the spins are parallel, the total spin is ($S=1$), and the state is a vector meson. This distinction is essential for understanding mass splitting in heavy mesons. Pseudoscalar and vector states with the same quark content would be degenerate if only a central potential were present, but experimentally they have different masses. Spin-dependent forces are therefore required. For heavy quarkonium, the most important examples are

$$cc\bar{c}: \eta_c, J/\psi, \psi'$$

and

$$bb\bar{b}: \eta_b, Y, Y'$$

The mass differences between these related states provide direct information about the spin-dependent part of the quark–antiquark interaction.

2.2 Heavy Mesons and Heavy Quarkonium

Heavy mesons are useful laboratories for studying the strong interaction because their spectra are sensitive to both perturbative and nonperturbative features of QCD. At short distances, the quark and antiquark interact through a Coulomb-like attraction associated with one-gluon exchange. At larger distances, confinement dominates and prevents the quarks from separating freely.

Charmonium and bottomonium are the most widely studied heavy quarkonium systems. Charmonium became especially important after the discovery of the (J/ψ), which provided strong evidence for the charm quark. Bottomonium later provided another heavy quarkonium system with even smaller relative velocities, making it especially suitable for nonrelativistic potential models.

Theoretical studies have approached heavy quarkonium in several ways. Eichten et al. [2] studied charmonium and compared the results with experiment. Martin [6] proposed a phenomenological potential capable of fitting several quark–antiquark spectra simultaneously. Quigg and Rosner [8] examined quarkonium level spacings and demonstrated the usefulness of logarithmic potentials. Lichtenberg et al. [5] tested static quark–antiquark potentials using bottomonium. Ebert et al. [3] developed a quark–antiquark potential including retardation and radiative contributions, while Ebert et

al. [10] studied hyperfine splitting and leptonic decay rates in heavy quarkonia.

These studies show that heavy meson spectroscopy can be understood as the result of three main physical effects: constituent quark masses, binding energy from the central potential, and spin-dependent corrections.

2.3 General Mass Formula

In a constituent potential model, the mass of a meson is written as

$$M = m_1 + m_2 + E_b$$

where (m_1) and (m_2) are the constituent quark and antiquark masses, and (E_b) is the binding energy of the system. In natural units, ($c=1$), so the binding-energy contribution can be added directly to the mass. In conventional units, the expression may be written as

$$M = m_1 + m_2 + \frac{E_b}{c^2}$$

For quarkonium systems, the quark and antiquark have the same constituent mass:

$$m_1 = m_2 = m_q$$

Therefore, the mass formula becomes

$$M = 2m_q + E_b$$

The binding energy is obtained from a bound-state equation. In the nonrelativistic approximation, this is usually the Schrödinger equation:

$$\left[-\frac{\hbar^2}{2\mu} \nabla^2 + V(r)\right]\psi(r) = E_b \psi(r)$$

where (μ) is the reduced mass,

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

For equal-mass quarkonium,

$$\mu = \frac{m_q}{2}$$

This simplification is one reason why charmonium and bottomonium are commonly used to test potential models.

2.4 Spin-Independent Quark–Antiquark Potential

The total quark–antiquark potential can be separated into spin-independent and spin-dependent parts:

$$V(r) = V_{SI}(r) + V_{SD}(r)$$

The spin-independent part determines the main binding and radial structure of the spectrum. A widely used form is the Cornell-type potential:

$$V_{SI}(r) = -\frac{a}{r} + br + c$$

The first term,

$$-\frac{a}{r}$$

is a short-range Coulomb-like term. It represents the attractive interaction between the quark and antiquark at small separations, where one-gluon exchange is dominant. The second term,

$$br$$

is a linear confining term. It increases with distance and reflects the physical fact that quarks are not observed as isolated free particles. The constant (c) shifts the energy scale and is usually fitted phenomenologically.

The Cornell potential is physically attractive because it combines two key features of QCD: asymptotic freedom at short distances and confinement at long distances. Although it is not a complete first-principles solution of QCD, it has been successful in describing heavy-quarkonium spectra [2, 3].

Other phenomenological potentials modify the radial dependence of the interaction while preserving the general idea of confinement. These include the Martin potential, Song–Lin potential, Turin potential, and logarithmic potential.

2.5 Spin-Dependent Potential

The spin-dependent part of the potential is responsible for fine and hyperfine splittings. It can be written schematically as

$$V_{SD}(r) = V_{SS}(r) + V_{LS}(r) + V_T(r)$$

where (V_{SS}) is the spin–spin interaction, (V_{LS}) is the spin–orbit interaction, and (V_T) is the tensor interaction.

The spin–spin interaction is especially important for (S)-wave states. It depends on the scalar product of the quark and antiquark spin operators:

$$\mathbf{S}_1 \cdot \mathbf{S}_2$$

For two spin-(1/2) particles,

$$\mathbf{S}_1 \cdot \mathbf{S}_2 = \frac{1}{2} [S(S+1) - S_1(S_1+1) - S_2(S_2+1)]$$

Since

$$S_1 = S_2 = \frac{1}{2}$$

the expectation value becomes

$$\mathbf{S}_1 \cdot \mathbf{S}_2 = -\frac{3}{4}$$

for (S=0), and

$$\mathbf{S}_1 \cdot \mathbf{S}_2 = \frac{1}{4}$$

for (S=1).

Thus, spin-zero and spin-one states receive different energy corrections. This explains why η_c and J/ψ have different masses, even though both are cc^- states. The same mechanism explains the splitting between η_b and Y .

A common feature of hyperfine splitting is that it decreases as the quark masses increase. This is because the spin–spin interaction is typically proportional to

$$\frac{1}{m_1 m_2}$$

Therefore, bottomonium hyperfine splittings are generally smaller than charmonium hyperfine splittings. This mass dependence is a key reason why spin-dependent potentials must be included when comparing different heavy meson families.

2.6 Phenomenological Potentials Used for Heavy Mesons

2.6.1 Martin Potential

The Martin potential is a phenomenological power-law potential introduced to describe several quark–antiquark spectra with a common functional form. Martin [6] proposed a simultaneous fit for bb^- , cc^- , ss^- , and cs^- systems. This made the potential useful for comparing different meson families within a single model.

A general power-law form may be written as

$$V(r) = a r^\nu + c$$

Where a, ν, c are fitted constants. The exponent ν determines how rapidly the potential grows with distance. The Martin potential is useful because it provides a flexible alternative to purely linear confinement.

2.6.2 Song–Lin Potential

The Song–Lin potential is another phenomenological interaction used in heavy meson spectroscopy. Song and Lin [7] proposed a potential form that modifies the radial dependence of the quark–antiquark interaction. Its inclusion allows researchers to test whether calculated mass spectra depend strongly on the exact shape of the confining potential.

If several potential forms give similar results, then the spectrum is likely controlled by general features such as short-distance attraction and long-distance confinement. If the results differ strongly, then the exact radial form of the potential becomes more important.

2.6.3 Turin Potential

The Turin potential, associated with Lichtenberg and collaborators, provides another static potential form for quark–antiquark systems. It can be written as

$$V(r) = -ar^{-3/4} + br^{3/4} + c$$

The negative-power term contributes mainly at short distances, while the positive-power term increases with (r) and contributes to confinement. Lichtenberg et al. [5] tested static quark–antiquark potentials using bottomonium, making this potential particularly relevant for ($b\bar{b}$) spectroscopy.

2.6.4 Logarithmic Potential

The logarithmic potential has the form

$$V(r) = b \ln(r) + c$$

This potential grows slowly with distance. Quigg and Rosner [8] showed that logarithmic potentials can be useful in describing quarkonium level spacings. The appeal of the logarithmic potential lies in its ability to produce approximately flavor-independent spacing patterns for heavy quarkonium states.

2.6.5 Cornell-Type Potential with Spin Corrections

A Cornell-type potential with spin corrections combines the central interaction with spin-dependent terms:

$$V(r) = -\frac{a}{r} + br + c + V_{SD}(r)$$

This form is widely used because it contains the essential physics of short-range one-gluon exchange, long-range confinement, and spin-induced splitting. Ebert et al. [3] improved this type of modeling by including retardation and radiative contributions, and Ebert et al. [10] further examined hyperfine splitting in heavy quarkonia.

2.7 Method of Calculation

The calculation of heavy meson masses can be organized into several steps.

First, the meson system is selected. For charmonium,

$$qq^- = cc^-$$

and for bottomonium,

$$qq^- = bb^-$$

Second, the constituent quark masses are chosen. These masses are effective parameters, not the same as current quark masses in the QCD Lagrangian. Constituent masses include part of the nonperturbative interaction energy associated with confinement.

Third, the interaction potential is specified. The selected potential may be Martin, Song–Lin, Turin, logarithmic, or Cornell-type. When spin corrections are included, the total potential becomes

$$V(r) = V_{central}(r) + V_{spin}(r)$$

Fourth, the radial Schrödinger equation is solved. For a central potential, the wave function is separated as

$$\psi(r, \theta, \phi) = R_{nl}(r)Y_{lm}(\theta, \phi)$$

Where $R_{nl}(r)$ is the radial wave function and Y_{lm} is the spherical harmonic. The radial equation gives the allowed binding energies.

Finally, the meson mass is calculated:

$$M = m_q + m_{q^-} + E_b$$

For quarkonium,

$$M = 2m_q + E_b$$

The calculated values are then compared with experimental values and previous theoretical predictions. This comparison is essential because potential models contain fitted parameters and must be tested against observed spectra.

3. RESULTS AND DISCUSSION

3.1 Physical Interpretation of the Results

The most important result of using spin-dependent potentials is the separation of states with different spin configurations. Without spin-dependent terms, pseudoscalar and vector mesons with the same radial and orbital quantum numbers would be nearly degenerate. This contradicts experiment.

For charmonium, the η_c and J/ψ states provide a clear example. Both contain cc^- , but η_c is a spin-singlet state, while J/ψ is a spin-triplet state. The mass difference between them is explained by the spin–spin part of the potential.

For bottomonium, the analogous splitting occurs between η_b and Y . Because the bottom quark is heavier than the charm quark, the hyperfine splitting is

smaller. This agrees with the expected dependence of the spin–spin interaction on $\frac{1}{m_1 m_2}$.

The success of spin-dependent potentials in describing these splittings supports the constituent quark model interpretation of heavy mesons. It also shows that even though potential models are phenomenological, they capture important physical effects of QCD.

3.2 Comparison with Previous Studies

The present framework is consistent with many earlier studies of heavy meson spectroscopy. Eichten et al. [2] demonstrated the effectiveness of potential models in charmonium. Martin [6] introduced a potential capable of fitting several quark–antiquark systems. Quigg and Rosner [8] studied level spacings and showed that logarithmic behavior could explain some regularities in quarkonium spectra. Song and Lin [7] proposed another potential form for quark–antiquark systems, while Lichtenberg et al. [5] tested static potentials with bottomonium.

Ebert and collaborators developed a series of relativistic quark-model studies that are especially important for heavy systems. Their work included heavy quarkonium mass spectra, retardation and radiative corrections, hyperfine splitting, and related applications to heavy hadrons [3, 10, 12–16]. These studies provide an important theoretical foundation for including spin-dependent terms in potential models.

Other works also support the broader framework. Rai et al. [4] studied (QQ^-) mesons in nonrelativistic QCD formalism. Ikhdair and Sever [11] investigated the (B_c^-) meson spectrum and hyperfine splitting using the shifted large- (N) expansion technique. Kumar and Monteiro [17] studied heavy quarkonia spectra and decays in a relativistic quark model. Bali et al. [18] examined static potentials from QCD simulations, while Jaczko and Durand [19] discussed the success of nonrelativistic potential models for relativistic quark–antiquark bound states.

Taken together, these studies show that potential models remain an important bridge between experimental spectroscopy and the underlying theory of strong interactions.

3.3 Limitations of the Model

Although spin-dependent potential models are useful, they have several limitations. First, they are phenomenological rather than exact solutions of QCD. Their parameters are usually fitted to experimental data, which means that predictive power depends on the quality and universality of the chosen parameters.

Second, relativistic effects may be important, especially for charmonium. Bottomonium is more suitable for nonrelativistic treatment because the bottom quark is heavier, but charmonium can require relativistic corrections.

Third, excited heavy meson states may be affected by open-flavor thresholds. For example, excited charmonium states can couple to DD^- channels, and

such effects may shift the masses away from simple potential-model predictions.

Fourth, spin-dependent interactions are often treated perturbatively. This may be adequate for low-lying heavy quarkonium states, but it can become less accurate for highly excited states or states near thresholds.

Finally, potential models usually focus on bound-state masses and may not fully describe decay widths, transition rates, or coupled-channel dynamics. More complete approaches may require lattice QCD, effective field theory, or relativistic Bethe–Salpeter methods.

4. CONCLUSION

In this article, the masses of heavy mesons were studied using spin-dependent quark–antiquark potentials. Heavy mesons were modeled as two-body systems composed of a quark and an antiquark. The total mass was calculated from the constituent quark masses and the binding energy obtained from the interaction potential.

Several potential forms were discussed, including the Martin, Song–Lin, Turin, logarithmic, and Cornell-type potentials. The Cornell-type form is especially important because it combines a short-range Coulomb-like interaction with a long-range confining term. However, a central potential alone cannot reproduce the observed fine and hyperfine structure of heavy mesons. Spin-dependent terms are therefore essential. The spin–spin interaction explains the mass splitting between pseudoscalar and vector states such as η_c and J/ψ , as well as η_b and Y . The smaller hyperfine splitting in bottomonium compared with charmonium follows naturally from the inverse dependence of the spin–spin interaction on the quark masses.

Overall, spin-dependent potential models provide a reliable and physically meaningful framework for heavy meson spectroscopy. Although they are not complete first-principles QCD calculations, they successfully capture the main features of heavy quarkonium: short-distance attraction, long-distance confinement, and spin-induced mass splitting. Future work can extend this model to higher orbital excitations, mixed-flavor systems such as (B_c) , coupled-channel effects, and relativistic corrections.

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Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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